Development of Probabilistic Stress Life Curves and Probabilistic Miner’s Damage Distribution Using Fatigue Testing Results

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OUTLINE

- Motivation and Objective
- Testing Data
- Probabilistic Stress Life (PSN)
- Stress Severity Factor (SSF)
- Random Damage Index (D)
- Discussion & Conclusions
- Current Work
Miner’s rule dictates that failure occurs when the damage index \((D)\) exceeds one. However, numerous comparisons with test results show that failure occurs for a range of damage index values, and the results are case and material dependent.
Objectives

1. Develop PSN curves that represents the testing variation.
2. Develop $\beta$ and $\theta$ from the testing data to be used in Jarfall’s Stress Severity Factor (SSF) method.
3. Determine probabilistic Miner’s damage distribution.

The probabilistic results will be used within a probabilistic simulation of fatigue failure of a general aviation structure.
Work Summary

Find the best Probabilistic SN Regression Fit

Develop Probabilistic SSF data

CA Testing

Testing

VA Testing

Damage Distributions

β, θ, SSF

Max. Stress
### Probabilistic SN

#### Available Data

<table>
<thead>
<tr>
<th>Coupon Configuration</th>
<th>Maximum Stress [KSI]</th>
<th>Number of Data Points</th>
<th>Mean Stress [KSI]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Open Hole</td>
<td>42, 32, 18, 12, 10, and 9.25</td>
<td>41</td>
<td>3</td>
</tr>
<tr>
<td>Open Hole</td>
<td>42, 32, 20, 18, 12.5, and 11.5</td>
<td>46</td>
<td>6</td>
</tr>
<tr>
<td>Hilok Filled Hole</td>
<td>42, 32, 24, 18, and 14</td>
<td>32</td>
<td>3</td>
</tr>
<tr>
<td>Hilok Filled Hole</td>
<td>42, 32, 30, 24, 21 and 16</td>
<td>37</td>
<td>6</td>
</tr>
<tr>
<td>Hilok 50 % Load Transfer</td>
<td>42, 32, 24, 15, and 8</td>
<td>37</td>
<td>3</td>
</tr>
<tr>
<td>Hilok 50 % Load Transfer</td>
<td>42, 32, 24, 15, and 11</td>
<td>45</td>
<td>6</td>
</tr>
</tbody>
</table>

Testing conducted by Wichita State University

This practice pertains only to S-N relationships that may be reasonable approximated by a straight line in log-log space. Do not use runouts for fitting

$$\log N = A + B(\log S)$$

$$B = \frac{\sum_{i=1}^{k} (X_i - \bar{X})(Y_i - \bar{Y})}{\sum_{i=1}^{k} (X_i - \bar{X})^2} \quad \hat{A} = \bar{Y} + \hat{B}\bar{X}$$

$$\bar{Y} = \hat{A} + \hat{B}\bar{X} \pm \sqrt{2F_p \sigma} \left[ \frac{1}{k} + \frac{(X - \bar{X})}{\sum_{i=1}^{k} (X_i - \bar{X})^2} \right]^{1/2}$$

$$\sigma^2 = \frac{\sum_{i=1}^{k} (Y_i - \bar{Y}_i)^2}{k - 2}$$
Probabilistic SN
Results Open Hole 3 KSI Mean Stress

--- 95% CB
Probabilistic SN
Hilok 50 Load Transfer 6 KSI Mean Stress

--- 95% CB
Probabilistic SN
Hilok 30 Load Transfer 6 KSI Mean Stress

--- 95% CB

--- 95% CB
Stress Severity Factor

Fatigue quality number based on the structural configuration

Hole Quality Factor, counts for the effect of the hole finishing.

\[ SSF = \alpha \beta \left( K_{tb} \times \Theta \times LT \times \frac{w}{d} + K_{tg} (1 - LT) \right) \]
The SSF concept can be extended for any discontinuity in the structure (Any discontinuity can be appraised by the SSF)
SSF
How to develop $\beta$

Because the LT = 0

$$\frac{\sigma(N_f)_{OH}}{\sigma(N_f)_{FH}} = \frac{SSF_{FH}}{SSF_{OH}}$$

Solve for $\beta_{FH}$

$\beta_{OH} = 1$
SSF
How to develop $\theta$

1. Hilok \textit{Load transfer}
2. Open Hole

\[
\frac{\sigma(N_f)_{OH}}{\sigma(N_f)_{FH}} = \frac{SSF_{FH}}{SSF_{OH}}
\]

\[
\frac{\sigma(N_f)_{OH}}{\sigma(N_f)_{FH}} = \alpha_{FH} \cdot \beta_{FH} \left\{ K_{tb} \cdot \theta \cdot LT \cdot \frac{w}{d} + K_{tb} (1 - LT) \right\} \quad \alpha_{OH} \cdot \beta_{OH} \cdot K_{tg}
\]

Solve for $\theta$
SSFβ and θ Results
Hilok 3 KSI Mean Stress (50 Load Transfer)
SSF = \alpha \beta \left( K_{tb} \times \theta \times LT \times \frac{w}{d} + K_{tg} \left(1 - LT\right) \right)

---

95% CB

### SSF

Hilok 3 KSI Mean Stress (50 Load Transfer)
Suppose we have the SN data for OH (SSF = 3) and we want to calculate the life for a structural configuration with SSF = 2.6

\[
\text{SSF Ratio} = \frac{SSFOH}{\text{Any SSF}} = \frac{3}{2.6} = 1.15
\]

\[
S_{\text{max}} \text{ Any SSF} = (\text{SSF Ratio}) \cdot S_{\text{max}} \text{ SSF OH}
\]

We can predict easily the life for any structural configuration knowing OH data and the SSF.
How to Use SSF to Predict Life

\[
\frac{S_{\text{max}}}{S_{\text{max}} @ \text{SSF} = 3} = \frac{3}{\text{Any SSF}}
\]

SSF Ratio

Stress [KSI] vs. Flight to Failure
Random Damage
Index Methodology

Usage Type
- Normal
- Aerobatic

Structural Details
- Fastener Type
- % Load Transfer (3 levels)
- 1g stress (3 levels)

Load Limit Factors
Flight Length and Velocity
Ground stress

Exceedance Curve
Sink Rate

Aerobatic - High 1g Stress - All Stages
Random Damage Index Methodology

Miner’s Rule

\[ d_{\text{current}} = d_{\text{previous}} + \frac{1}{N_{f_j}} \]

Yes Next Cycle

No, Store Damage Index

Fastener Type: A/B
% Load Transfer: x/y/z
1g stress (ksi) = i/j/k

Failure Damage Index D

Repeat N test M Simulations
## Random Damage Index Results

<table>
<thead>
<tr>
<th>Spectrum</th>
<th>Severity</th>
<th>Configuration</th>
<th>Mean Damage Index</th>
<th>Coefficient of Variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal</td>
<td>High (9 KSI)</td>
<td>Open Hole</td>
<td>0.7248</td>
<td>0.113</td>
</tr>
<tr>
<td>Normal</td>
<td>Medium (7 KSI)</td>
<td>Open Hole</td>
<td>0.8774</td>
<td>0.190</td>
</tr>
<tr>
<td>Normal</td>
<td>Low (5 KSI)</td>
<td>Open Hole</td>
<td>0.7281</td>
<td>0.228</td>
</tr>
<tr>
<td>Normal</td>
<td>High (9 KSI)</td>
<td>50% Load Transfer</td>
<td>5.7379</td>
<td>0.483</td>
</tr>
<tr>
<td>Normal</td>
<td>Medium (7 KSI)</td>
<td>50% Load Transfer</td>
<td>2.2056</td>
<td>0.437</td>
</tr>
<tr>
<td>Normal</td>
<td>Low (5 KSI)</td>
<td>50% Load Transfer</td>
<td>Coupon did not fail during testing</td>
<td></td>
</tr>
<tr>
<td>Aerobatic</td>
<td>High (6 KSI)</td>
<td>Open Hole</td>
<td>0.8942</td>
<td>0.101</td>
</tr>
<tr>
<td>Aerobatic</td>
<td>Medium (4.5 KSI)</td>
<td>Open Hole</td>
<td>0.9151</td>
<td>0.131</td>
</tr>
<tr>
<td>Aerobatic</td>
<td>Low (3 KSI)</td>
<td>Open Hole</td>
<td>0.7495</td>
<td>0.135</td>
</tr>
<tr>
<td>Aerobatic</td>
<td>High (6 KSI)</td>
<td>50% Load Transfer</td>
<td>2.4138</td>
<td>0.225</td>
</tr>
<tr>
<td>Aerobatic</td>
<td>Medium (4.5 KSI)</td>
<td>50% Load Transfer</td>
<td>4.3957</td>
<td>0.468</td>
</tr>
<tr>
<td>Aerobatic</td>
<td>Low (3 KSI)</td>
<td>50% Load Transfer</td>
<td>Coupon did not fail during testing</td>
<td></td>
</tr>
</tbody>
</table>
Random Damage Index Results

Normal Usage, High Severity, and Open Hole

Weibull 3 Parameter
Random Damage Index Results

Acrobatic Usage, High Severity, and Open Hole

Weibull 3 Parameter
Random Damage Index Results

Normal Usage, Low Severity, and Open Hole

Weibull 3 Parameter
Random Damage Index Results

Acrobatic Usage, Low Severity, and Open Hole

Weibull 3 Parameter
Random Damage Index Results

Normal Usage, High Severity, and Hilok 50%

Weibull 3 Parameter
Random Damage Index Results

Acrobatic Usage, High Severity, and Hilok 50%

Weibull 3 Parameter
Probalistic methodology and computer code were developed so that a probabilistic damage index can be calculated.

Results confirm that failure does not always occur when Miner’s coefficients reaches a value equal to one.

The results for $D$ were best represented by a Weibull distribution.
Conclusions & Discussion

- The mean values ranged from approximately 0.72 for low severity Normal usage open hole coupon to 5.73 for high severity normal usage 50% load transfer coupon, and from 0.74 for low severity Acrobatic open hole coupon to 4.39 for high severity acrobatic usage 50% load transfer.
Current Work

- Methodology implementation in SMART (SMall Aircraft Risk Technology) software to do risk assessment of general aviation.
- Implement Polynomial regression for the tested data.
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Questions?
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Thanks!